**Homework 6, due 11/5**

In these problems, $\omega_1, \omega_2 \in \mathbf{C}$ are such that $\tau = \omega_2/\omega_1 \in H$. Let

$$\wp(z) = \frac{1}{z^2} + \sum_{(m,n)\in\mathbf{Z}^2\setminus\{(0,0)\}} \left[\frac{1}{(z+m\omega_1+n\omega_2)^2} - \frac{1}{(m\omega_1+n\omega_2)^2}\right].$$

1. Show that near $z=0$ the function $\wp$ has expansion

$$\wp(z) = \frac{1}{z^2} + \frac{1}{20}g_2 z^2 + \frac{1}{28}g_3 z^4 + O(z^6),$$

where

$$g_2 = 60 \sum_{(m,n)\neq(0,0)} \frac{1}{(m\omega_1+n\omega_2)^4},$$
$$g_3 = 140 \sum_{(m,n)\neq(0,0)} \frac{1}{(m\omega_1+n\omega_2)^6}.$$

2. Show that

$$(\wp')^2 = 4\wp^3 - g_2\wp - g_3,$$

and

$$\wp'' = 6\wp^2 - \frac{1}{2}g_2.$$

3. Show that

$$g_2 = -4(e_1e_2 + e_1e_3 + e_2e_3),$$
$$g_3 = 4e_1e_2e_3$$

in terms of the values $e_1 = \wp(\omega_1/2), e_2 = \wp(\omega_2/2), e_3 = \wp\big((\omega_1+\omega_2)/2\big)$ at the half periods.